Vehicle Routing Using Simulated Annealing with Cauchy Acceptance Criterion

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ABSTRACT

This study compares Boltzmann and Cauchy's acceptance criterion in a simulated annealing algorithm to solve vehicle routing problems (VRP). Also, the initial states setting compare between randomly setting and using cluster information by k-means algorithm. The embedded cluster structure is introduced in the neighborhood search criterion to explore nearly optimal. The proposed algorithm is tested on different scales (15, 30, 50, and 100 nodes). The results show that the Cauchy SA algorithm can escape local minima and has efficiency converge in terms of shorter iterations. In addition, the guided cluster neighborhood search has the potential to further improved cost.

MOTIVATION

Algorithm Simulated annealing

Initialize the parameters: temperature T, reduction factor c
While termination criterion is not satisfied **do**
  for number of candidate *(generate by neighborhood search)*
    Select a new solution
    if $f(x_{candidate}) < f(x_0)$
      $f_{new} = f(x_{candidate})$
    else
      $\Delta f = f(x_{candidate}) - f(x_0)$
      random r(0,1)
      if $r < \exp(-\Delta f/T)$
        $f_{new} = f(x_{candidate})$
      else
        $f_{new} = f(x_0)$
      end if
    end if
  $f = f_{new}$
  Decrease the temperature: T = c T
  end for
end while

Key component of SA [1]

- Acceptance criterion
- Neighborhood search

The Cauchy distribution implies an occasional acceptance to escape local minima [2]

The cluster method can decompose search space.

METHOD

Optimization method:

Solving VRP by Simulated Annealing

Objective function: $\sum_{i,j \in A} c_{ij} x_{ij}$

Minimize travel distance

A is set of arcs

c_{ij} is cost of travel over arc

$x_{ij} \in \{0,1\}$

Each vehicle deliver to customers and fulfill all demand with limit capacity.

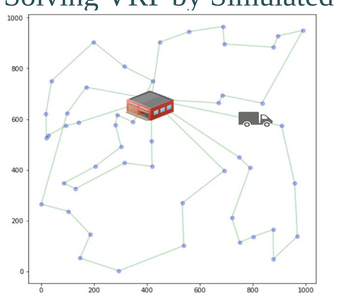

Figure 1: Example VRP model

Simulated Annealing:

- **Acceptance criterion**

Boltzmann equation $exp(-\frac{\Delta f}{T})$

Cauchy equation $\frac{T}{T^2 + \Delta f^2}$

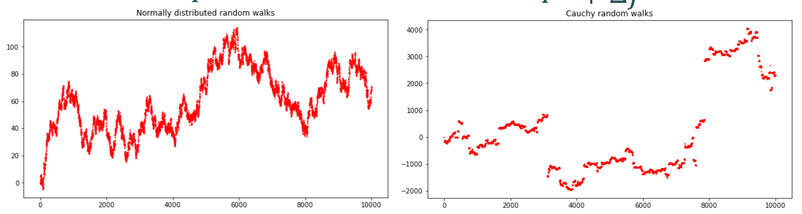

Figure 2: Comparison of free space random walks

- **Neighborhood search**

Use swap operator and K means method to guide neighborhood search

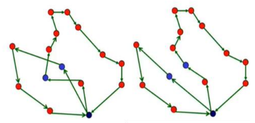

Figure 3: 1-1 swap operator

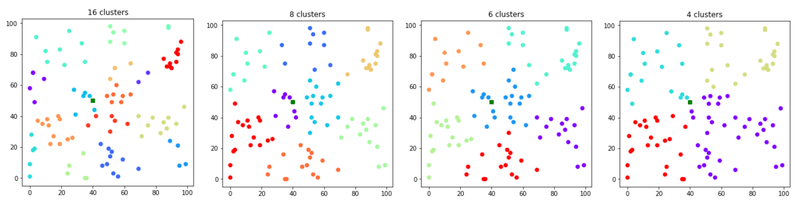

Figure 4: Guided cluster in neighborhood search (16, 8, 6, 4 clusters respectively)

Evaluate Model: Minimum Cost and number of iterations

RESULTS

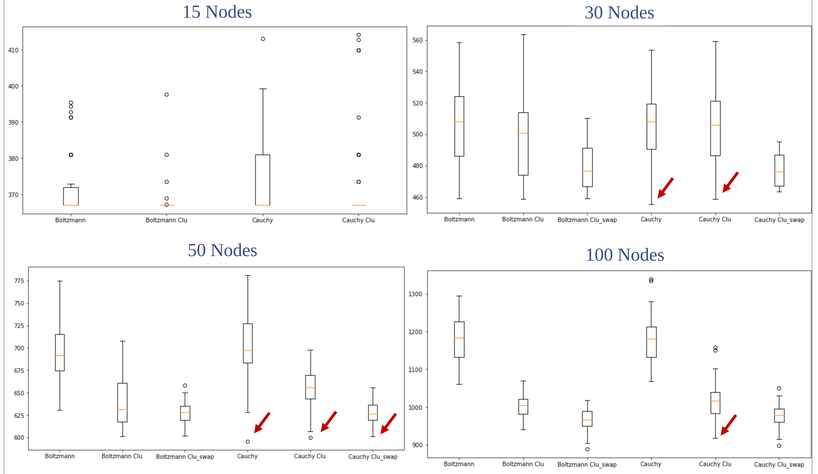

Figure 5: Comparison cost (Randomly initial, Cluster initial, Guided search)

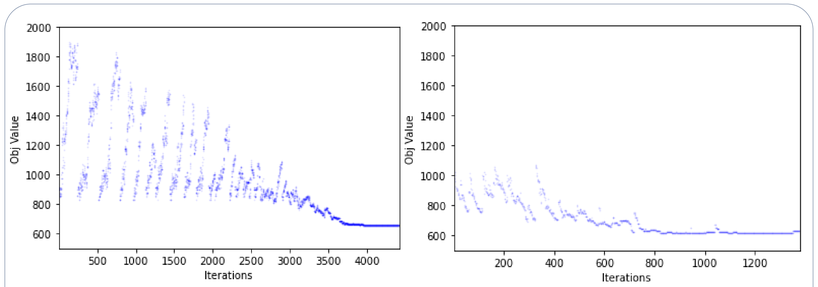

Figure 6: Accepted cost each iteration between Boltzmann (left) and Cauchy (right)

	Initial random neighbor				Initial Cluster				Cluster neighborhood search			
	15	30	50	100	15	30	50	100	15	30	50	100
Boltzmann	367.0	459.1	630.8	1061.6	367.0	458.9	601.1	941.7	367.0	459.1	601.6	**889.3**
Cauchy	367.0	**455.4**	**595.5**	1068.8	367.0	458.9	600.1	918.0	367.0	463.7	601.4	898.2

Table I: Compare overall minimum result

CONCLUSIONS

- By provided cluster information in the initial state, both equations can take advantage of the information. However, Boltzmann requires a suitable initial temperature, but Cauchy much less cautious in temperature settings.
- The Cauchy equation can escape local minima by occasional long jump. Therefore, the Cauchy has better performance in the minimum result.
- For the neighborhood search criterion, the embedded cluster can guide random walk to search for a better solution.
- By compared the number of iterations, the Cauchy equation can provide a solution in a short time.

REFERENCES

[1] Franzin, Alberto, and Thomas Stützle. "Revisiting simulated annealing: A component-based analysis." Computers & operations research 104 (2019): 191-206.

[2] Szu, Harold, and Ralph Hartley. "Fast simulated annealing." Physics letters A 122.3-4 (1987): 157-162.