Non-Linear Connectivity Analysis in Brain Using Maximal Correlation and Bayesian Network

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Abstract

In this work, we aim to propose a method to detect the non-linear correlation among different areas of the brain and visualize it. We first extend the original maximal correlation to the random process scenario, theoretically. And we use the partial correlation and the MAP principle to construct a Gaussian Bayesian network to visualize the non-linear connectivity. The result shows that our non-linear correlation can discover the functional connectivity that the linear correlation cannot detect.

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Introduction

Different parts of the brain are cooperating closely instead of handling specific tasks individually. This insight inspires biologists and neuroscientists to consider the connectivity in the brain, especially, the non-linear functional connections in our brain.

In this work, we focus on:

- Measure and visualize the non-linear connectivity in the brain.
- Propose a new non-linear correlation for two random processes.

Maximal Correlation [1,2]

Given two discrete time random process $X[t]$, $t \in \mathcal{T}$ and $Y[s]$, $s \in \mathcal{S}$. Neither $X[t]$ nor $Y[s]$ have a constant realization with probability 1. Denote the sets of non-constant square integrable functions by $\mathcal{F}: \mathbb{R} \to \mathbb{R}$ and $\mathcal{G}: \mathbb{R} \to \mathbb{R}$, respectively. The maximal correlation for $X[t]$ and $Y[s]$ is defined as

$$
\begin{aligned}
R(X,Y) &= \max_{f_t \in \mathcal{F}, g_s \in \mathcal{G}} \frac{1}{|\mathcal{T}\,\|\,\mathcal{S}|} \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \mathbb{E}^2\left[f_t(X[t]) g_s(Y[s]) \right] \\
s.t. \quad & \mathbb{E}\left[f_t(X[t]) \right] = \mathbb{E}\left[g_s(Y[s]) \right] = 0 \\
& var(f_t(X[t])) = var(g_s(Y[s])) = 1, \\
& \text{for all } t \in \mathcal{T} \text{ and } s \in \mathcal{S}
\end{aligned}
$$

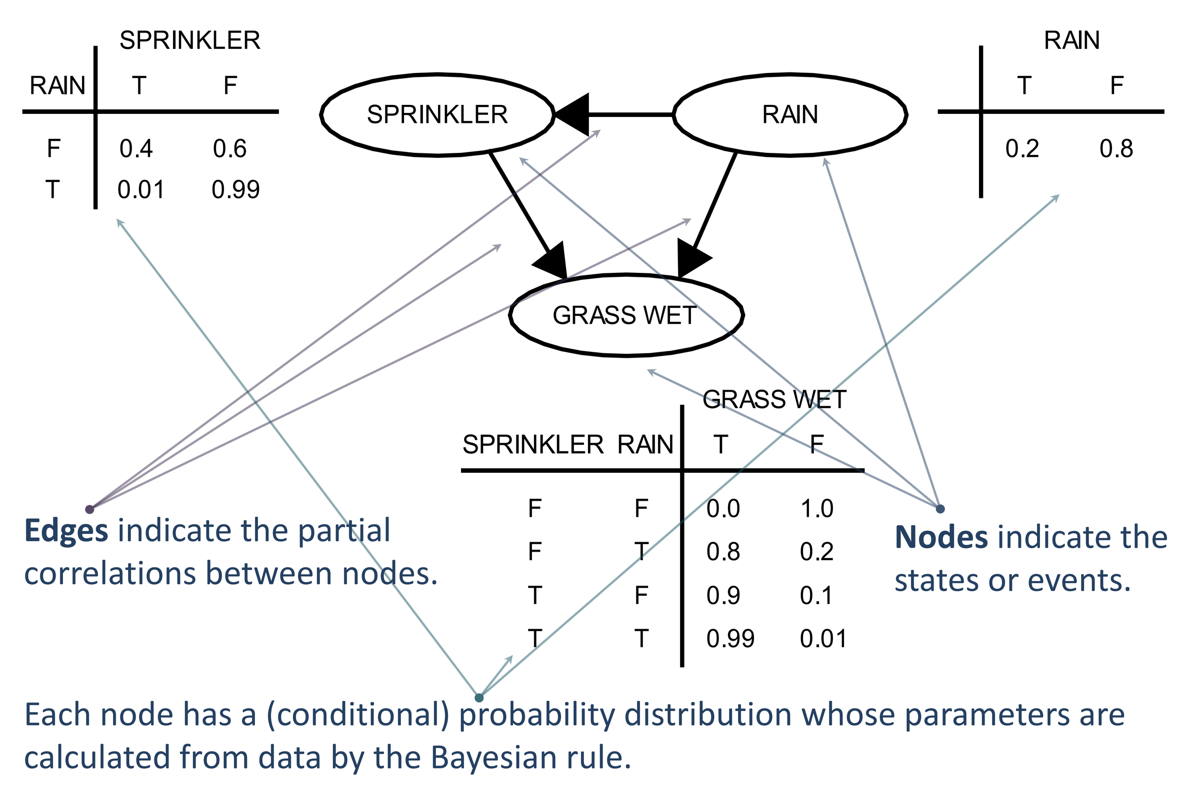

Figure 1. An Illustration of Bayesian Network. [3]

Bayesian Network

The Bayesian network is a probabilistic graph model that uses a directed acyclic graph to represent a set of variables or states and the conditional dependence, and all of the unknown parameters are calculated by the Bayesian rule.

Data & Experiment

The dataset used to calculate the correlation matrix is the TUH EEG Seizure Corpus dataset [4].

We use both of the Pearson correlation and the maximal correlation to evaluate the brain connectivity. The result of the correlation matrix is shown in the Figure 3.

The dataset we used to calculate the Bayesian network is EEG dataset in the UCL Machine Learning Repository.[5]

- **Partial correlation** is used to construct the Bayesian Network.
- Our Bayesian network follows a **Gaussian network** assumption.
- The values of unknown parameters are calculated by the **MAP principle**.

The results of the structure of the Bayesian network are shown in figure 2. The nodes indicate the sensors of the EEG device. The edge between a pair of nodes indicates that there is a partial correlation between them.

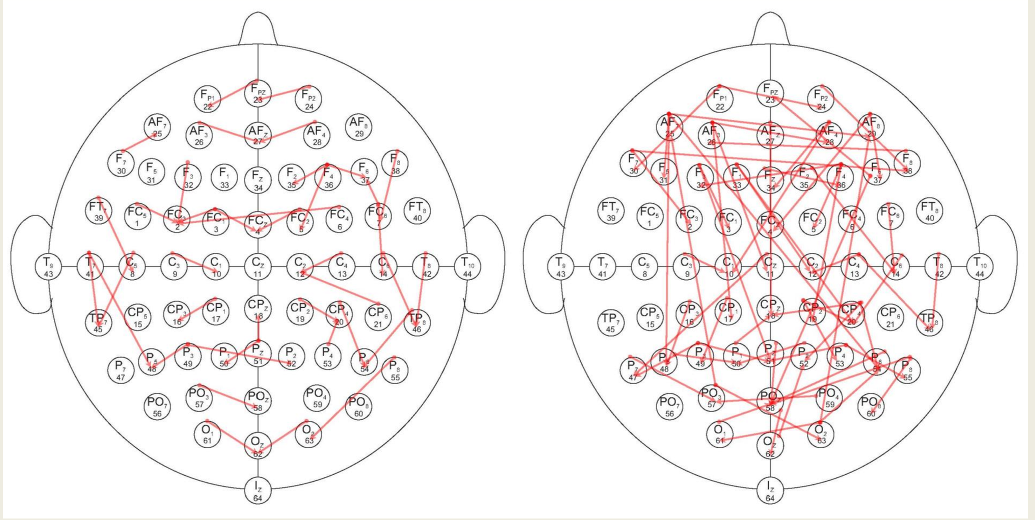

Figure 2. Bayesian Network Structure.
Linear correlation(Left), non-linear correlation(right)

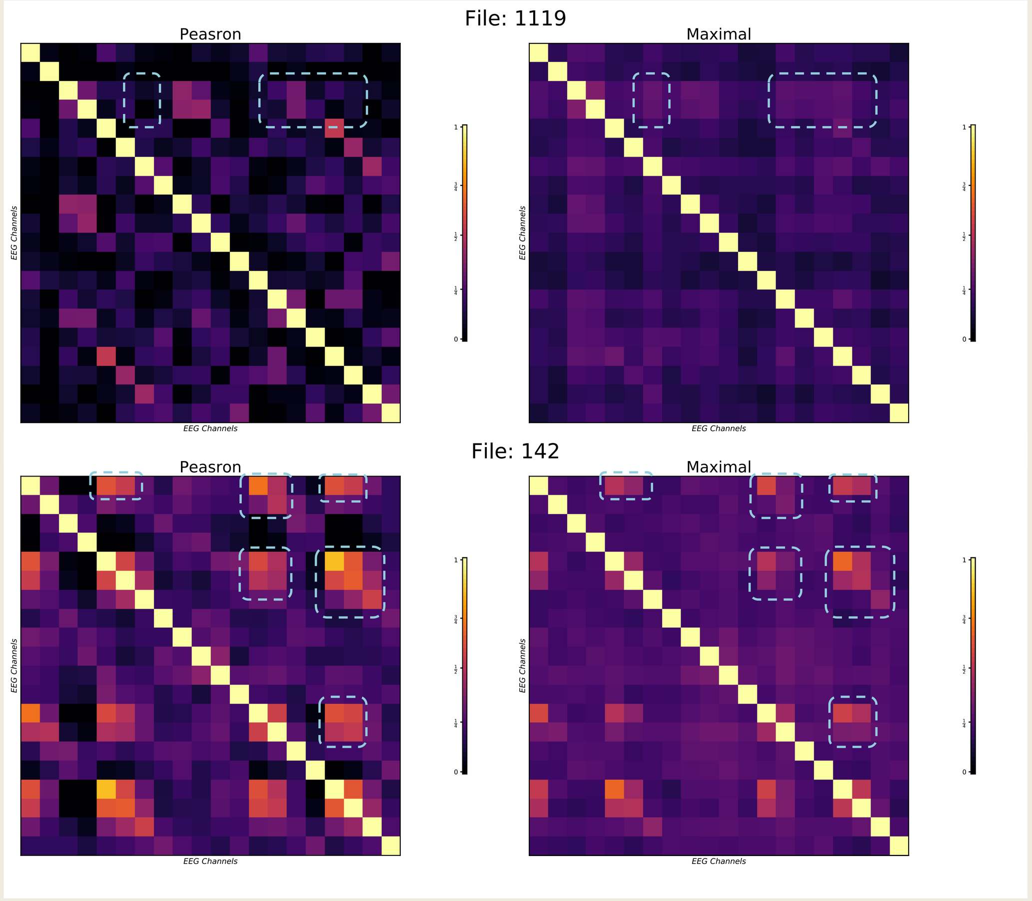

Figure 3. Correlation Matrices.
Linear correlation(Left), non-linear correlation(right)

Conclusions

The correlation matrix figure and the Bayesian Network shows:

- Non-linear correlation can discover the dependence which may be neglected by the linear correlation. Non-linear correlation can also detect the linear dependence between two signals.
- Linear correlation does well in finding the local physical connection. Non-linear correlation can discover functional relationships between two distant regions.

Reference

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